

Gauging the Heisenberg algebra of special quaternionic manifolds

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Abstract

We show that in $N = 2$ supergravity, with a special quaternionic manifold of (quaternionic) dimension $h_1 + 1$ and in the presence of h_2 vector multiplets, a $h_2 + 1$ dimensional abelian algebra, intersecting the $2h_1 + 3$ dimensional Heisenberg algebra of quaternionic isometries, can be gauged provided the $h_2 + 1$ symplectic charge-vectors V_I , have vanishing symplectic invariant scalar product $V_I \times V_J = 0$. For compactifications on Calabi-Yau three-folds with Hodge numbers (h_1, h_2) such condition generalizes the half-flatness condition as used in the recent literature. We also discuss non-abelian extensions of the above gaugings and their consistency conditions.

1. The Heisenberg algebra

It is well known [1, 2] that the moduli space of a Calabi–Yau compactification of Type II string theory is a product of a special quaternionic manifold \mathcal{M}_{SQ} of quaternionic dimension $h_1 + 1$ and a special Kähler manifold \mathcal{M}_{SK} of complex dimension h_2 where $h_1 = h_{(2,1)}$, $h_2 = h_{(1,1)}$ for Type IIA and the reverse for Type IIB.

The special quaternionic geometry has some general properties [2, 3, 4], i.e. the $2h_1 + 3$ coordinates which describe $2h_1 + 2$ R–R scalar fields and the a scalar field dual to the antisymmetric tensor field $B_{\mu\nu}$ [5], parametrize, in the “solvable description” of the manifold [6], a Heisenberg algebra of the form:

$$\begin{aligned} [X^\Lambda, Y_\Sigma] &= \delta^\Lambda_\Sigma \mathcal{Z} ; \quad \Lambda = 0, \dots, h_1, \\ [X^\Lambda, X^\Sigma] &= [Y_\Lambda, Y_\Sigma] = [X^\Lambda, \mathcal{Z}] = [Y_\Lambda, \mathcal{Z}] = 0. \end{aligned} \quad (1)$$

In Calabi–Yau compactifications the generators X^Λ, Y_Σ in (1) are parametrized by the RR real scalars which in Type IIA come from the internal components of the complex 3–form $A_{(3)}$ [7, 8, 9]:

$$\{\tilde{\zeta}_\Lambda, \zeta^\Lambda\} \rightarrow \{A_{ijk}, A_{i\bar{j}\bar{k}}, A_{\bar{i}\bar{j}\bar{k}}, A_{\bar{i}jk}, \} . \quad (2)$$

while Type IIB they originate from the 2–form and 4–form cohomology:

$$\{\tilde{\zeta}_\Lambda, \zeta^\Lambda\} \rightarrow \{C, C_{i\bar{j}\bar{l}k}, C_0, C_{i\bar{j}}\} , \quad (3)$$

where C is the dual of $C_{\mu\nu}$.

The universal hypermultiplet contains, besides the dilaton and the a field which parametrizes the generator \mathcal{Z} in (1), the $\Lambda = 0$ component of the above coordinates, namely $\{\text{Re}A_{ijk}, \text{Im}A_{ijk}\}$ in Type IIA and $\{C_0, C\}$ in Type IIB. In each case such multiplets parametrize $\mathcal{M}_U = \text{SU}(1, 2)/\text{U}(2) \subset \mathcal{M}_{SQ}$. Under the group of motions generated by the Heisenberg algebra the scalar fields $\tilde{\zeta}_\Lambda, \zeta^\Lambda$ transform as follows [2]:

$$\begin{aligned} \delta\zeta^\Lambda &= u^\Lambda \\ \delta\tilde{\zeta}_\Lambda &= v_\Lambda \\ \delta a &= w + u^\Lambda \tilde{\zeta}_\Lambda - v_\Lambda \zeta^\Lambda. \end{aligned} \quad (4)$$

Noting that $\delta(\tilde{\zeta}_\Lambda \zeta^\Lambda) = u^\Lambda \tilde{\zeta}_\Lambda + v_\Lambda \zeta^\Lambda$ we may redefine a in such a way that one of the two scalar–dependent terms in δa is eliminated.

2. The gaugings

Let us define a gauge algebra through the following infinitesimal field transformations:

$$\delta A_\mu^I = \partial_\mu \lambda^I ,$$

$$\begin{aligned}
\delta\zeta^\Lambda &= a_I^\Lambda \lambda^I, \\
\delta\tilde{\zeta}_\Lambda &= b_{I\Lambda} \lambda^I, \\
\delta a &= c_I \lambda^I + (a_I^\Lambda \tilde{\zeta}_\Lambda - b_{I\Lambda} \zeta^\Lambda) \lambda^I,
\end{aligned} \tag{5}$$

where $I = 0, \dots, h_2$, h_2 being the number of vector multiplets. Note that no relation exists between h_1, h_2 so that the above algebra is not in general contained in the Heisenberg algebra.

The covariant derivatives read:

$$\begin{aligned}
D_\mu \zeta^\Lambda &= \partial_\mu \zeta^\Lambda - a_I^\Lambda A_\mu^I, \\
D_\mu \tilde{\zeta}_\Lambda &= \partial_\mu \tilde{\zeta}_\Lambda - b_{I\Lambda} A_\mu^I, \\
D_\mu a &= \partial_\mu a - (a_I^\Lambda \tilde{\zeta}_\Lambda - b_{I\Lambda} \zeta^\Lambda) A_\mu^I - c_I A_\mu^I.
\end{aligned} \tag{6}$$

One can verify that:

$$\begin{aligned}
\delta(D_\mu \zeta^\Lambda) &= \delta(D_\mu \tilde{\zeta}_\Lambda) = 0, \\
\delta(D_\mu a) &= (a_I^\Lambda D_\mu \tilde{\zeta}_\Lambda - b_{I\Lambda} D_\mu \zeta^\Lambda) \lambda^I,
\end{aligned} \tag{7}$$

where in order to derive the last equation we required requires the following condition:

$$c_{IJ} \equiv b_{I\Lambda} a_J^\Lambda - b_{J\Lambda} a_I^\Lambda = 0, \tag{8}$$

which we shall characterize in the sequel as a “cocycle” condition of the Lie algebra. If we consider $\{a_J^\Lambda, b_{I\Lambda}\}$ to be the $2h_1 + 2$ components of a symplectic vector V_I , condition (8) can be rephrased as the vanishing of the symplectic scalar product $V_I \times V_J = 0$. Such condition is also equivalent to the closure of the abelian gauge algebra whose generators $\{T_I\}$ are:

$$T_I = b_{I\Lambda} X^\Lambda + a_I^\Lambda Y_\Lambda + c_I \mathcal{Z} ; \quad [T_I, T_J] = 0. \tag{9}$$

3. Gauging of special quaternionic σ -model

There is an elegant way of writing the RR scalars in the quaternionic manifold in terms of the symplectic section:

$$Z = \begin{pmatrix} \zeta^\Lambda \\ \tilde{\zeta}_\Lambda \end{pmatrix}, \tag{10}$$

and the symplectic (symmetric) matrix \mathcal{M} [10]:

$$\mathcal{M} = \begin{pmatrix} \mathbb{1} & -\text{Re}(\mathcal{N}) \\ 0 & \mathbb{1} \end{pmatrix} \begin{pmatrix} \text{Im}(\mathcal{N}) & 0 \\ 0 & \text{Im}(\mathcal{N})^{-1} \end{pmatrix} \begin{pmatrix} \mathbb{1} & 0 \\ -\text{Re}(\mathcal{N}) & \mathbb{1} \end{pmatrix}. \tag{11}$$

Indeed the kinetic term [2] is given by:

$$K_{a\bar{b}} \partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} - \frac{1}{4\phi^2} (\partial\phi)^2 - \frac{1}{4\phi^2} (\partial a - Z \times \partial Z)^2 - \frac{1}{2\phi} \partial_\mu Z \mathcal{M} \partial^\mu Z. \quad (12)$$

Note that invariance under the Heisenberg algebra with symplectic parameters:

$$\Theta = \begin{pmatrix} u^\Lambda \\ v_\Sigma \end{pmatrix}, \quad (13)$$

is manifest since

$$\delta a = w + \Theta \times Z; \quad \delta Z = \Theta \Rightarrow da - Z \times dZ \text{ invariant}. \quad (14)$$

The gauging of the non linear σ -model goes as follows. We consider an abelian $h_2 + 1$ dimensional gauge group whose embedding in the Heisenberg algebra is described by $h_2 + 1$ symplectic charge-vectors

$$V_I = \begin{pmatrix} a_I^\Lambda \\ b_{I\Lambda} \end{pmatrix}, \quad (15)$$

and whose connection U is expressed in terms of the $h_2 + 1$ vector fields A_μ^I as follows:

$$U = A^I V_I, \quad (16)$$

$$\delta U = d\Theta, \quad (17)$$

Θ being now $\Theta = \lambda^I V_I$, where λ^I are the gauge parameters. The covariant derivative of Z is then

$$DZ = dZ - U, \quad (18)$$

and the covariant derivative of a reads

$$Da = da - U \times Z, \quad (19)$$

since $\delta a = \Theta \times Z$.

If we transform Da we obtain

$$\delta(Da) = \Theta \times dZ - U \times \Theta = \Theta \times (dZ + U), \quad (20)$$

which is not $\Theta \times DZ$ since $DZ = dZ - U$. Therefore closure implies $\Theta \times U = 0$ which is equivalent to condition (8) since:

$$\Theta \times U = 2 a_{[I}^\Lambda b_{J]\Lambda} \lambda^I A^J. \quad (21)$$

If $\Theta \times U = 0$ we can write the RR sector of the gauged Lagrangian

$$-\frac{1}{4\phi^2} (Da - Z \times DZ)^2 - \frac{1}{2\phi} D_\mu Z \mathcal{M} D^\mu Z. \quad (22)$$

Upon addition of the minimal coupling $\partial a - c_I A^I$ to the covariant derivative of a the vector boson mass matrix M_{IJ}^2 will read:

$$M_{IJ}^2 = \frac{1}{2\phi^2} (c_I - 2 Z \times V_I) (c_J - 2 Z \times V_J) + \frac{1}{\phi} V_I \mathcal{M} V_J. \quad (23)$$

4. Non-abelian gauging

Let us now see what are the requirements which have to be satisfied in order to embed a non-abelian gauge algebra in the Heisenberg algebra.

Quite generally we introduce a non-abelian gauge algebra defined by:

$$[T_I, T_J] = f_{IJ}^K T_K. \quad (24)$$

Using the embedded expression for the gauge algebra generators given in equation (9), the embedding condition

$$f_{IJ}^K T_K = c_{IJ} \mathcal{Z}, \quad (25)$$

implies the following relations:

$$f_{IJ}^K c_K = c_{IJ}, \quad (26)$$

$$f_{IJ}^K b_{K\Lambda} = 0, \quad (27)$$

$$f_{IJ}^K a_K^\Lambda = 0. \quad (28)$$

In terms of the Lie algebra cohomology equation (26) means that c_{IJ} is a non trivial cocycle of the gauge algebra, while (27) and (28) imply that $b_{I\Lambda}$ and a_I^Λ are coboundaries. When $c_{IJ} = 0$ the cohomology is trivial and we are in the case of the abelian gauge algebra discussed in the previous section. Since the algebra (25) contains a central charge it is non-semisimple and according to a theorem of Lie algebra cohomology we may have a non trivial cocycle c_I in the adjoint representation of the algebra (this would be impossible if the gauge algebra were semisimple since in that case the only non trivial cocycle should be in the trivial representation of the algebra). In fact a solution of conditions (26),(27),(28) may be found as follows. We first consider the case in which $h_2 + 1 = 2h_1 + 3$, so that the number of vector matches the dimension of the Heisenberg algebra. The gauge generators T_I decompose in the following way:

$$\{T_I\} = \{T_\Lambda, T^\Lambda, T_0\}. \quad (29)$$

The charge matrices are chosen to be

$$\begin{aligned} b_{0\Lambda} &= b_{\Sigma\Lambda} = 0; \quad b^\Sigma_\Lambda = b_\Lambda \delta^\Sigma_\Lambda, \\ a_0^\Lambda &= a^{\Sigma\Lambda} = 0; \quad a_\Sigma^\Lambda = a^\Lambda \delta_\Sigma^\Lambda, \end{aligned} \quad (30)$$

$$(31)$$

The cocycle condition (26) becomes

$$c_0 f_\Lambda^{0\Sigma} = (b_\Lambda a^\Lambda) \delta_\Lambda^\Sigma, \quad (32)$$

with no summation over the index Λ . Conditions (27), (28) are manifestly verified. If $h_2 > 2(h_1 + 1)$ we may apply the above construction to $2h_1 + 3$ vectors while the remaining $h_2 - 2(h_1 + 1)$ vectors stay spectators. Viceversa, if $h_2 < 2(h_1 + 1)$ we can select a Heisenberg subalgebra with $\bar{h}_1 = \frac{h_2}{2} - 1$ and apply to it the construction described above.

5. Gauging and half-flatness

Let us consider Calabi–Yau compactifications on a *half-flat* manifold [13]. For the kind of manifolds considered in [11, 12], in the absence of fluxes, we have the following couplings in Type IIA and IIB theories

$$\text{IIA} \quad a_I^\Lambda = 0 \ ; \ b_{I\Lambda=0} = \epsilon_I \ ; \ (0 \text{ otherwise}) \ , \quad (33)$$

$$\text{IIB} \quad a_I^\Lambda = 0 \ ; \ b_{I=0\Lambda} = \epsilon_\Lambda \ ; \ (0 \text{ otherwise}) \ , \quad (34)$$

and the cocycle condition (8) is identically satisfied (recall that, according to our notations, in Type IIA $I = 0, \dots, h_{1,1}$ and $\Lambda = 0, \dots, h_{2,1}$ while in Type IIB $I = 0, \dots, h_{2,1}$, $\Lambda = 0, \dots, h_{1,1}$). Note that we use the same symbols to denote the charges a_I^Λ , $b_{I\Lambda}$ in Type IIA and IIB theories although they are described in the two cases by different matrices with different dimensions. If we turn on a NS 3-form flux in Type IIA theory we have $a_{I=0}^\Lambda = p^\Lambda \neq 0$ and $b_{I=0\Lambda} = q_\Lambda \neq 0$, and then, on a half-flat manifold we should also have a non vanishing a_I^Λ since the cocycle condition requires:

$$a_I^\Lambda b_{J\Lambda} = a_J^\Lambda b_{I\Lambda} \Rightarrow q_0 a_I^0 = p^0 \epsilon_I \ . \quad (35)$$

On the Type IIB side, if we turn on an electric NS 3-form flux we get a covariant derivative of the type [14]:

$$D_\mu \tilde{\zeta}_0 = \partial_\mu \tilde{\zeta}_0 - q_I A_\mu^I \ , \quad (36)$$

where q_I is the *electric* flux $b_{I\Lambda=0}$. If Type IIB background is half-flat [11] we also have $b_{I=0\Lambda} = \epsilon_\Lambda \neq 0$. In this case, as expected, $b_{00} = \epsilon_0 = q_0$. For the *magnetic* NS 3-form flux the correspondence is non-local.

The abelian gauging of the Heisenberg algebra discussed in the previous sections therefore generalizes the results on flux-compactifications on half-flat manifolds as discussed in the literature [11, 12], to arbitrary values of I , Λ . Consistency always requires in the “dual theories” the cocycle condition to be satisfied:

$$a_{[I}^\Lambda b_{J]\Lambda} = 0 \ ; \ (I, J = 0, \dots, h_2 \ ; \ \Lambda = 0, \dots, h_1) \ .$$

Mirror symmetry on the other hand implies

$$b^{(B)}_{I\Lambda} = (b^{(A)T})_{I\Lambda} \ ; \ (I = 0, \dots, h_{2,1} \ ; \ \Lambda = 0, \dots, h_{1,1}) \ ,$$

In Type IIA theory we can interpret the parameters a_I^Λ , $b_{I\Lambda}$ of the gauging in terms of the following deformation of the Calabi–Yau cohomology [12, 11]:

$$\begin{aligned} d\alpha_\Lambda &= b_{i\Lambda} \omega^i \ ; \ d\beta^\Lambda = a_i^\Lambda \omega^i \ , \\ d\omega_i &= a_i^\Lambda \alpha_\Lambda - b_{i\Lambda} \beta^\Lambda \ , \\ \omega_i &\in H^{(1,1)} \ ; \ \omega^i \in H^{(2,2)} \ ; \ i = 1 \dots, h_{1,1} \ . \end{aligned} \quad (37)$$

in the presence of a non trivial NS flux:

$$\hat{H}_{(3)} = dB_{(2)} + d(b^i \omega_i) - a_0^\Lambda \alpha_\Lambda + b_{0\Lambda} \beta^\Lambda. \quad (38)$$

Integrability on the cohomology side gives:

$$d\omega^i = 0 ; \quad d^2\omega_i = -(a_i^\Lambda b_{j\Lambda} - a_j^\Lambda b_{i\Lambda}) \omega^j = 0, \quad (39)$$

while on the NS flux it implies

$$d\hat{H}_{(3)} = 0 \Rightarrow (a_0^\Lambda b_{j\Lambda} - a_j^\Lambda b_{0\Lambda}) \omega^j = 0. \quad (40)$$

Conditions (39),(40) are equivalent to the cocycle condition (8).

One can show that the definition of the a_I^Λ , $b_{I\Lambda}$ given in (37) is consistent. Indeed, for instance, on one hand we can write:

$$\int d\alpha_\Lambda \wedge \omega_i = - \int \alpha_\Lambda \wedge (a^\Sigma_j \alpha_\Sigma - b_{j\Sigma} \beta^\Sigma) = b_{j\Lambda}, \quad (41)$$

while on the other hand we have:

$$b_{i\Lambda} \int \omega^i \wedge \omega_j = b_{j\Lambda}. \quad (42)$$

By performing a compactification on such *half-flat* Calabi–Yau in the presence of a NS flux we have

$$\begin{aligned} d\hat{A} &= dA_0, \\ d\hat{B}_{(2)} &= dB_{(2)} + db^i \wedge \omega_i - b^i (a_i^\Lambda \alpha_\Lambda - b_{i\Lambda} \beta^\Lambda), \\ d\hat{C}_{(3)} &= d\tilde{A}^i \wedge \omega_i - (\zeta^\Lambda b_{i\Lambda} - \tilde{\zeta}_\Lambda a_i^\Lambda) \omega^i - (d\zeta^\Lambda - a_i^\Lambda \tilde{A}^i) \wedge \alpha_\Lambda + \\ &\quad (d\tilde{\zeta}_\Lambda - b_{i\Lambda} \tilde{A}^i) \wedge \beta^\Lambda, \\ \hat{F}_{(4)} &= d\hat{C}_{(3)} + \hat{H}_{(3)} \wedge A^0 = dA^i \wedge \omega_i - b^i dA^0 \wedge \omega_i - (d\zeta^\Lambda - a_I^\Lambda A^I) \wedge \alpha_\Lambda + \\ &\quad (d\tilde{\zeta}_\Lambda - b_{I\Lambda} A^I) \wedge \beta_\Lambda - (\zeta^\Lambda b_{i\Lambda} - \tilde{\zeta}_\Lambda a_i^\Lambda) \omega^i + dB_{(2)} \wedge A^0, \\ A^i &= \tilde{A}^i + b^i A^0. \end{aligned} \quad (43)$$

So we obtain the correct gauging of the Ramond isometries. The covariant derivative of the scalar field a dual to $B_{\mu\nu}$ is obtained from the topological term in the IIA ten-dimensional action as in [12].

6. Conclusions

In this note we have studied the gauging of the Heisenberg algebra which is common to all special quaternionic manifolds, and proved that, for an abelian gauge algebra, it

requires a vanishing cocycle condition (i.e. that a certain Lie algebra cocycle be trivial). This gauge algebra, as it appears in Calabi–Yau compactification with fluxes or/and half-flat manifolds, corresponds to the gauging of isometries acting on RR scalars and the (dual of) the NS 2-form. The symplectic structure exhibited by the RR scalars embedded in a special quaternionic manifold suggests the general form of the gauging and a mirror relation when switching to the Heisenberg algebra of the mirror theory. It is suggestive that, if this is done, new couplings are predicted that do not usually appear in the perturbative formulation of Type IIA and Type IIB theories. The general gauging of the Heisenberg algebra also induces a scalar potential which, in some particular cases, has been studied in [11] and [12], and whose general properties are under investigation.

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